SOLARTOWER: a numerical code for the calculation of the solar energy collected in a solar tower power plant.

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FIG. 1: Solar tower power plant.

I. ABSTRACT

The numerical code SOLARTOWER is presented in the this paper. It is written in Fortran77 and it allows the calculation of the solar energy collected to the receiver in a solar tower power plant, given an arbitraty solar field of heliostats.

II. INTRODUCTION

The solar field of a central receiver system, or power tower, is made up of several hundred or even thousand mirrors, called heliostats, placed around a receiver at the top of a central tower, as shown in Fig. 1. A computer controls each of these two-axis tracking heliostats with a tracking error of less than a fraction of a degree to ensure that the reflected sunlight focuses directly on the tower receiver, where an absorber is heated up to temperatures of about 1000 C by the concentrated sunlight. Air or molten salt transports the heat and a gas or steam turbine drives an electrical generator that transforms the heat into electricity.

The location of the mirrors around the tower is a crucial aspect of the system and it depends on many factors. In fact, the field of heliostats suffers losses caused by shading and blocking by neighboring heliostats. In particular, shading occurs when a heliostat casts its shadow on another heliostat located behind it, while blocking occurs when a heliostat in front of another heliostat blocks the reflected suns energy on its way to the receiver. Of course, the amount of energy collected to the receiver is a a function of the distance of the each heliostat from both the receiver and the other heliostats of the field. In the present paper it is described the numerical code SOLARTOWER which calculates the energy collected from a solar tower plant for any arbitrary disposition of the mirrors around the receiver. The rest of the paper is organized as follows: in Sec. III the input data of the code are indicated, in Sec. IV the calculation of the zenith and azimuth angles are described, in Sec. V it is illustrated how the orientation of the heliostats is performed, in Sec. VI the shading and blocking effects are described, in Sec. VII the strategy used for the calculations of the shading and blocking contributions on each panel by means of change of system coordinates and numerical integration is described, and finally, in Sec. VIII some examples of the solar tower performance are given.

III. DESCRIPTION OF THE INPUT DATA OF THE CODE

The code SOLARTOWER needs as input data the geographic coordinates (latitude and longitude in degrees and primes) of the site of interest, the direct normal radiation data, in Wh/m^2, for a given hour of a given day of the julian year of interest, and the three coordinates corresponding to the location of the center of each heliostat in the solar field. Each heliostat is assumed of squared shape with side dimension, l_p, also given as input data, in meters. The cartesian coordinate system is assumed with the positive x-axis corresponding to the EAST direction, and the y-axis corresponding to the NORTH direction (see Fig. 2). The receiver is located at the center of the coordinate system, at a height, l_r, also given in meters as input data.

IV. CALCULATION OF THE SUN POSITION

The solar energy incoming on the earth surface is a complex function of the latitude and, of course, it varies depending on the sun position in the sky. Such position is defined by two angles, called zenith and azimuth, and in the following indicated as α_z and ϕ, respectively. In particular, α_z is the angle between the sun ray and the normal to the earth surface, while ϕ is the angle of the projection of the sun ray on the earth surface with the SOUTH direction. Here it is considered the rule for which $\phi > 0$ for projections in the SOUTH/EAST quadrant (morning), $\phi < 0$ in the SOUTH/WEST quadrant (evening), and $\phi = 0$ at noon (considering the Northern emisphere above the Tropic of Cancer). The zenith and azimuth angles depend on the latitude, the hour and the day, and are calculated as indicated below.

First of all it is calculated the solar declination, that is the angle between the plane perpendicular to the line Earth-Sun and the terrestrial axis of rotation,

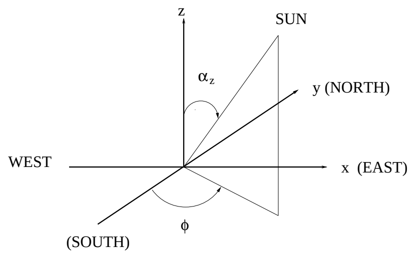

FIG. 2: System of coordinate used. The receiver is located at height $z = l_r$ in the origin.

$$\begin{aligned} \delta_s &= 0.006918 - 0.399912 \times \cos(\tau_d) + \\ &+0.070257 \times \sin(\tau_d) - 0.006758 \times \cos(2\tau_d) + \\ &+0.000907 \times \sin(2\tau_d) - 0.002697 \times \cos(3\tau_d) + \\ &+0.001480 \times \sin(3\tau_d), \end{aligned} \tag{1}$$

where $\tau_d = 2\pi\frac{day-1}{365.0}$

Then, it is calculated the hour angle, ω, which represents the sun position with respect to the NORTH/SOUTH axis, measured in the equatorial plane, and it depends from the solar hour, LT. In order to the have the local hour, if the UT (universal time) hour is given (corresponding to Greenwich time), it is necessary to add (Longlocal-Longsm)/15, where Longlocal is the longitude of the observation point, in degrees, and Longsm is the longitude of the reference meridian in the time zone of interest.

$$\omega = -15.0(LT - 12.0) \tag{2}$$

Finally, α_z and ϕ are given as follows:

$$\begin{aligned} \alpha &= \mathrm{arcos}[\sin(\delta_\mathrm{s})\sin(\mathrm{lat}\pi/180.0) + \\ &+cos(\delta_s)cos(\mathrm{lat}\pi/180.0)\cos(\omega\pi/180.0)] \end{aligned} \tag{3}$$

$$h = \frac{\pi}{2} - \alpha \tag{4}$$

$$\gamma = \mathrm{arcos}\left[\frac{\sin(\mathrm{lat}\pi/180.0)\sin(\mathrm{h}) - \sin(\delta_\mathrm{s})}{\cos(\mathrm{lat}\pi/180.0)\cos(\mathrm{h})}\right] \tag{5}$$

dove lat is the latitude, in degrees, and h is the altitude.

V. HELIOSTAT ORIENTATION

Heach heliostat of the solar field is oriented in the correct way, that is, in order to reflect the sun ray to the receiver, by means of two rotations around two fixed axes passing across the center of the panel and parallel to the x and z axes.

First of all, it is calculated the normalized vector, $\overrightarrow{v(i)}$, from the center of the panel to the receiver: We call $P_x(i)$, $P_y(i)$, and $P_z(i)$, the cartesian coordinates of the center of the heliostat i, and $r_x(i) = |P_x(i)|$, $r_y(i) = |P_y(i)|$, and $r_z(i) = h_r - P_z(i)$, then four cases we have to destinguish :

- panel located in the NORTH-EAST quadrant ($P_x(i) \geq 0$ and $P_y(i) \geq 0$):

$$\begin{aligned} v_x(i) &= \frac{-r_x(i)}{\sqrt{r_x(i)^2 + r_y(i)^2 + r_z(i)^2}} \\ v_y(i) &= \frac{-r_y(i)}{\sqrt{r_x(i)^2 + r_y(i)^2 + r_z(i)^2}} \\ v_z(i) &= \frac{r_z(i)}{\sqrt{r_x(i)^2 + r_y(i)^2 + r_z(i)^2}} \end{aligned} \tag{6}$$

- panel located in the NORTH-WEST quadrant ($P_x(i) < 0$ and $P_y(i) \geq 0$):

$$\begin{aligned} v_x(i) &= \frac{r_x(i)}{\sqrt{r_x(i)^2 + r_y(i)^2 + r_z(i)^2}} \\ v_y(i) &= \frac{-r_y(i)}{\sqrt{r_x(i)^2 + r_y(i)^2 + r_z(i)^2}} \\ v_z(i) &= \frac{r_z(i)}{\sqrt{r_x(i)^2 + r_y(i)^2 + r_z(i)^2}} \end{aligned} \tag{7}$$

- panel located in the SOUTH-EAST quadrant ($P_x(i) \geq 0$ and $P_y(i) < 0$):

$$\begin{aligned} v_x(i) &= \frac{-r_x(i)}{\sqrt{r_x(i)^2 + r_y(i)^2 + r_z(i)^2}} \\ v_y(i) &= \frac{r_y(i)}{\sqrt{r_x(i)^2 + r_y(i)^2 + r_z(i)^2}} \\ v_z(i) &= \frac{r_z(i)}{\sqrt{r_x(i)^2 + r_y(i)^2 + r_z(i)^2}} \end{aligned} \tag{8}$$

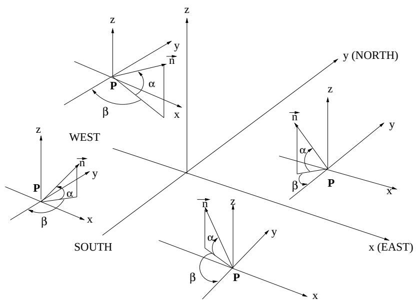

FIG. 3: α and β angles.

- panel located in the SOUTH-WEST quadrant ($P_x(i) < 0$ and $P_y(i) < 0$):

$$
\begin{aligned}
v_x(i) &= \frac{r_x(i)}{\sqrt{r_x(i)^2 + r_y(i)^2 + r_z(i)^2}} \\
v_y(i) &= \frac{r_y(i)}{\sqrt{r_x(i)^2 + r_y(i)^2 + r_z(i)^2}} \\
v_z(i) &= \frac{r_z(i)}{\sqrt{r_x(i)^2 + r_y(i)^2 + r_z(i)^2}}
\end{aligned} \tag{9}
$$

Then, it is calculated the normalized vector from the sun to the center of the panel, $\overrightarrow{s}$:

$$
\begin{aligned}
s_x &= \frac{sin(\phi)}{\sqrt{sin(\phi)^2 + cos(\phi)^2 + tg\left(\frac{\pi}{2} - \alpha_z\right)^2}} \\
s_y &= \frac{-cos(\phi)}{\sqrt{sin(\phi)^2 + cos(\phi)^2 + tg\left(\frac{\pi}{2} - \alpha_z\right)^2}} \\
s_z &= \frac{tg\left(\frac{\pi}{2} - \alpha_z\right)}{\sqrt{sin(\phi)^2 + cos(\phi)^2 + tg\left(\frac{\pi}{2} - \alpha_z\right)^2}}
\end{aligned} \tag{10}
$$

Finally, the normal to the panel plane, $\overrightarrow{n(i)}$ is obtained from $\overrightarrow{v(i)}$ and $\overrightarrow{s(i)}$ as it follows:

$$
\begin{aligned}
n_x(i) &= \frac{v_x(i) + s_x}{2} \\
n_y(i) &= \frac{v_y(i) + s_y}{2} \\
n_z(i) &= \frac{v_z(i) + s_z}{2}
\end{aligned} \tag{11}
$$

At this point, two new angles must be defined, α_i and β_i (see. Fig. 3).

In particular, $\alpha_i = \mathrm{arcos}(\sqrt{\mathrm{n_x(i)^2 + n_y(i)^2}})$, represents the angle that $\overrightarrow{n(i)}$ forms with the ground plane, while β_i, is the angle between the projection of $\overrightarrow{n(i)}$ on the ground plane with the

NORTH-SOUTH direction, and it is positive when $\overrightarrow{n(i)}$ is directed at SOUTH. Two cases must be marked:

- $n_x(i) \leq 0$ and $n_y(i) \leq 0$ or $n_x(i) \geq 0$ and $n_y(i) \leq 0 \Longrightarrow \beta_i = \arcsin\left(\frac{n_x(i)}{\sqrt{n_x(i)^2+n_y(i)^2}}\right)$
- $n_x(i) \geq 0$ and $n_y(i) \geq 0$ or $n_x(i) \leq 0$ and $n_y(i) \geq 0 \Longrightarrow \beta_i = \pi - \arcsin\left(\frac{n_x(i)}{\sqrt{n_x(i)^2+n_y(i)^2}}\right)$

It is assumed that, before orientation, each heliostat is in a starting position with its plane parallel to the xz cartesian plane, with its nomal vector oriented towards SOUTH. Therefore, given the two rotation matrices,

$$R(x) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & cos(\alpha_i) & sin(\alpha_i) \\ 0 & -sin(\alpha_i) & cos(\alpha_i) \end{vmatrix}.$$

and

$$R(z) = \begin{vmatrix} cos(\beta_i) & -sin(\beta_i) & 0 \\ sin(\beta_i) & cos(\beta_i) & 0 \\ 0 & 0 & 1 \end{vmatrix}.$$

,

the cartesian coordinates of the four edges of each heliostat, A^+, A^-, B^+, and B^- are obtained as it follows:

$$\begin{bmatrix} A^+_x(i) \\ A^+_y(i) \\ A^+_z(i) \end{bmatrix} = R(x) \times R(z) \times \begin{bmatrix} -l_p \\ 0 \\ l_p \end{bmatrix} + \begin{bmatrix} P_x(i) \\ P_y(i) \\ P_z(i) \end{bmatrix}$$

$$\begin{bmatrix} A^-_x(i) \\ A^-_y(i) \\ A^-_z(i) \end{bmatrix} = R(x) \times R(z) \times \begin{bmatrix} -l_p \\ 0 \\ -l_p \end{bmatrix} + \begin{bmatrix} P_x(i) \\ P_y(i) \\ P_z(i) \end{bmatrix}$$

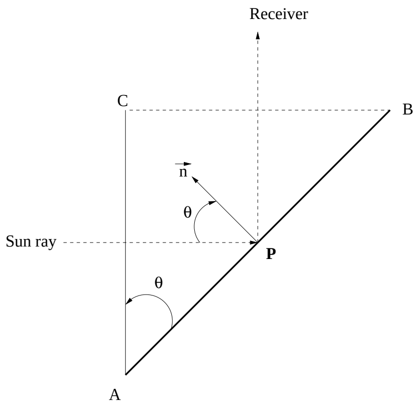

FIG. 4: Representation of the coseno effect. $\bar{(}AB$ is the panel lenght and $\bar{AC} = \bar{AB}\cos(\theta)$ is the projection of $\bar{AB}$ along the sun rays direction.

$$\begin{bmatrix} B_x^+(i) \\ B_y^+(i) \\ B_z^+(i) \end{bmatrix} = R(x) \times R(z) \times \begin{bmatrix} l_p \\ 0 \\ l_p \end{bmatrix} + \begin{bmatrix} P_x(i) \\ P_y(i) \\ P_z(i) \end{bmatrix}$$

$$\begin{bmatrix} B_x^-(i) \\ B_y^-(i) \\ B_z^-(i) \end{bmatrix} = R(x) \times R(z) \times \begin{bmatrix} l_p \\ 0 \\ -l_p \end{bmatrix} + \begin{bmatrix} P_x(i) \\ P_y(i) \\ P_z(i) \end{bmatrix}$$

At this point it is introduced the concept of the 'cosine effect' or 'cosine loss', representing the difference between the amount of energy falling on a surface pointing at the sun, and the surface of the heliostat. For each oriented panel this energy loss depends on the angle θ_i between $\overrightarrow{n(i)}$ and $\overrightarrow{s}$. In particular, if E_S is the direct normal solar irradiation, expressed in Wh/m^2, the amount of solar energy collected on a heliostat is $E_S \times \cos\theta_i$. A representation of the cosine effect is depicted in Fig.4, where $\bar{AB}^2\cos\theta$ is the projection of the surface $\bar{AB}^2$ on the sun ray direction.

Once the coordinates of the four edges of each heliostat have been calculated, it can be written the equation of the plane passing through them. In particular, it is calculated the equation of the plane passing through the center of the panel $\overrightarrow{P(i)}$ and two of its edges, $\overrightarrow{A^+(i)}$ and $\overrightarrow{A^-(i)}$, respectively. The four coefficients of the plane equations,

$$a(i)x + b(i)y + c(i)z + d = 0 \tag{12}$$

can be readly obtained:

$$
\begin{aligned}
a(i) &= P_y(i)A_z^+(i) + P_z(i)A_y^-(i) + A_y^+(i)A_z^-(i) + \\
&\quad -A_y^-(i)A_z^+(i) - P_y(i)A_z^-(i) - P_z(i)A_y^+(i) \\
b(i) &= -[P_x(i)A_z^+(i) + P_z(i)A_x^-(i) + A_x^+(i)A_z^-(i) + \\
&\quad -A_z^+(i)A_x^-(i) - P_x(i)A_z^-(i) - P_z(i)A_x^+(i)] \\
c(i) &= P_x(i)A_y^+(i) + P_y(i)A_x^-(i) + A_x^+(i)A_y^-(i) + \\
&\quad -A_y^+(i)A_x^-(i) - P_x(i)A_y^-(i) - P_y(i)A_x^+(i) \\
d(i) &= -[P_x(i)A_y^+(i)A_z^-(i) + P_y(i)A_z^+(i)A_x^-(i) + \\
&\quad P_z(i)A_x^+(i)A_y^-(i) - A_x^-(i)A_y^+(i)P_z(i) + \\
&\quad -P_x(i)A_y^-(i)A_z^+(i) - A_x^+(i)P_y(i)A_z^-(i)]
\end{aligned} \tag{13}
$$

VI. SHADING AND BLOCKING

In order to evaluate the fraction of solar energy which arrives to the receiver, it is necessary to follow the path of the sun rays. The first part of their trajectory is from the sun to the heliostat, and the showdows caused on a panel k from the other panels i is called 'shading effect'. For each panel k, the shading is calculated by considering the intersection of the sun rays passing through the four edges of the other panels i with the plane of the panel k.

Of course not all panels give a contribution to the shading, therefore a selection criterion must be considered.

It is then considered a straight line passing through the center of the panel k and orthogonal to the sun rays, whose equation is:

$$
y = mx + n; \tag{14}
$$

where $m = tg(\phi)$ and $n = P_y(k) - mP_x(k)$. If the panel i is below this line, that is,

$$
P_y(i) < [tg(\phi)P_x(i) + P_y(k) - tg(\phi)P_x(k)], \tag{15}
$$

such panel must be considered.

Given the cosine directors of the sun rays:

$$\alpha = -\sin\phi \tag{16}$$

$$\beta = \cos\phi \tag{17}$$

$$\gamma = \cos\alpha_{\mathrm{z}} \tag{18}$$

the intersection of the sun rays passing through the eadges $\overrightarrow{A^{+}(i)}$, $\overrightarrow{A^{-}(i)}$, $\overrightarrow{B^{+}(i)}$, and $\overrightarrow{B^{-}(i)}$ with the plane of the panel k gives origin to correspondly four points, $\overrightarrow{A_s^{+}(k)}$, $\overrightarrow{A_s^{-}(k)}$, $\overrightarrow{B_s^{+}(k)}$, and $\overrightarrow{B_s^{-}(k)}$ whose coordinates are explicitly given below for $\overrightarrow{A_s^{+}(k)}$:

$$A^{+}_{x_s}(k) = A^{+}_{x}(i) + \alpha t \tag{19}$$

$$A^{+}_{y_s}(k) = A^{+}_{y}(i) + \beta t \tag{20}$$

$$A^{+}_{z_s}(k) = A^{+}_{z}(i) + \gamma t \tag{21}$$

where

$$t = -\left[\frac{a(k)A^{+}_{x}(i) + b(k)A^{+}_{y}(i) + c(k)A^{+}_{z}(i) + d(k)}{a(k)\alpha + b(k)\beta + c(k)\gamma}\right] \tag{22}$$

Analogously, we obtain the components for the other three intersection points.

The sun rays reflected from the panel k can be blocked from the panels i positioned at radius, $R(i) < R(k)$. This is the so called ”blocking effect”. In this case, the intersection of the reflection straightline passing through the four edges of the blocking panel i with the plane of the panel k must be calculated:

$$\begin{aligned}
A^{+}_{z_b}(k) &= \frac{1}{C} \times \left[\frac{-a(k)P_x(k)A^{+}_{z}(i)}{h_r - P_z(k)} - a(k)A^{+}_{x}(i) + \right. \\
&\quad \left. -b(k)A^{+}_{y}(i) - \frac{b(k)P_y(k)A^{+}_{z}(i)}{h_r - P_z(k)} - d(k)\right] \\
A^{+}_{x_b}(k) &= \frac{-P_x(k)}{h_r - P_z(k)}\left[A^{+}_{z_b}(k) - A^{+}_{z}(i)\right] + A^{+}_{x}(i) \\
A^{+}_{y_b}(k) &= \frac{-P_y(k)}{h_r - P_z(k)}\left[A^{+}_{z_b}(k) - A^{+}_{z}(i)\right] + A^{+}_{y}(i)
\end{aligned} \tag{23}$$

where

$$C = -\frac{a(k)P_x(k)}{h_r - P_z(k)} - \frac{b(k)P_y(k)}{h_r - P_z(k)} + c(k) \tag{24}$$

Analougsly the points $\overrightarrow{A_b^{-}(k)}$, $\overrightarrow{B_b^{+}(k)}$, and $\overrightarrow{B_b^{-}(k)}$ are calculated.

VII. CHANGE OF THE REFERENCE SYSTEM OF COORDINATES

The next step performed by the code is a change of coordinate system, that is, from that with the origin at the receiver to that with the origin at the centre of the heliostat, in order to easly calculate the fraction of solar energy sent to the receiver. Therefore, into a loop over all the heliostats of the solar field, for the four edges of each heliostat a double rotation exactly inverse to that which positioned the heliostats from the starting position (in the plane xz) to the right orientation (see $R(x)$ and $R(z)$ of Sec. V) is performed with

$$R'(x) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & cos(\alpha_i) & -sin(\alpha_i) \\ 0 & sin(\alpha_i) & cos(\alpha_i) \end{vmatrix} .$$

and

$$R'(z) = \begin{vmatrix} cos(\beta_i) & sin(\beta_i) & 0 \\ -sin(\beta_i) & cos(\beta_i) & 0 \\ 0 & 0 & 1 \end{vmatrix} .$$

Then, for example, the point $A^+(i)$ is converted into $A^+_{new}(i)$ as:

$$\begin{bmatrix} A^+_{x_{new}}(i) \\ A^+_{y_{new}}(i) \\ A^+_{z_{new}}(i) \end{bmatrix} = R(x) \times R(z) \times \begin{bmatrix} A^+_x(i) - P_x(i) \\ A^+_y(i) - P_y(i) \\ A^+_z(i) - P_z(i) \end{bmatrix}$$

Similar transformations are given for $A^-(i)$, $B^+(i)$, $B^-(i)$, $A^+_s(i)$, $A^-_s(i)$, $B^+_s(i)$, $B^-_s(i)$, $A^+_b(i)$, $A^-_b(i)$, $B^+_b(i)$, and $B^-_b(i)$.

In the new coordinate system the four edges of the panel i are in the plane xz with coordinates $(-l_p/2, -l_p/2)$, $(l_p/2, -l_p/2)$, $(-l_p/2, l_p/2)$, and $(l_p/2, l_p/2)$.

The calculation of the fraction of energy (taking account of both blocking and shadowing effects) is performed by numerical integration on each panel: a grid in x and z is constructed on the panel surface and it is verified if each small square of such subdivision contributes to collect energy or not.

VIII. SOME EXAMPLES OF A SOLAR TOWER PERFORMANCE

In this sections some considerations are given on the solar tower performances.

A. Test case 1: Circular crown of heliostats around the receiver with fixed sun position

It is considered in this subsection the case of a solar field composed of 1398 heliostats placed into a circular crown around the receiver, as shown in Fig. 5, with minimum radius of 30 m and maximum radius of 300 m. The panel size is 10×10 m^2, and the panels are located at chessboard, with a distance between adjacent panels of 14.142 m, so that they occupy about one half of the land surface, with a constant density. A series of simulations has been performed varying the receiver height with the sun position fixed at $\alpha_z = 0^o$ and $\phi = 0^o$. In particular, the receiver heigth, h_r, considered are 25, 50, 100, and 150 m, respectively, and the direct normal solar irradiation is 1 kWh/m^2. As attained and shown in Fig. 6, the solar energy collected at the receiver increases with the receiver height. In particular, at $h_r = 25$ m and 50 m shading and blocking involve all the heliostats of the field. As long as the receiver height increases, these effects involve a less number of panels, as evident for $h_r = 100$ or 150 m, where a larger number of panels closer to the receiver affects the only cosine effect. The total energy collected at the receiver, as function of the receiver height is shown in Fig.7, where the calculated points have been also interpolated with a cubic function. The goodness of the interpolation is demonstrated from the fact that the results of two other calculations of the total energy corresponding to $h_r = 75$ and 125 m, respectively, lies on the fitting curve.

In an analougus series of simulations, differing only in the fact that the panel size has been reduced to $1{\times}1$ m^2, and therefore the total number of heliostats increased to 139940, in order to mantain exactly the same land coverage, calculated energies have been compared. The results indicate that there is not significant difference in the collected energies, as shown in Fig. 8

B. Test case 2: Comparison of performance between a solar tower plant and a parabolic trough plant

The performances of the solar tower plant (ST) described in the previous Subsec. VIII A are compared with that of a parabolic trough power plant (PT). In particular, the solar field is supposed localized at Macchiareddu (Cagliari-Italy) (latitude: 39^o 15′ North, Longitude: 8^o 57′ East), and

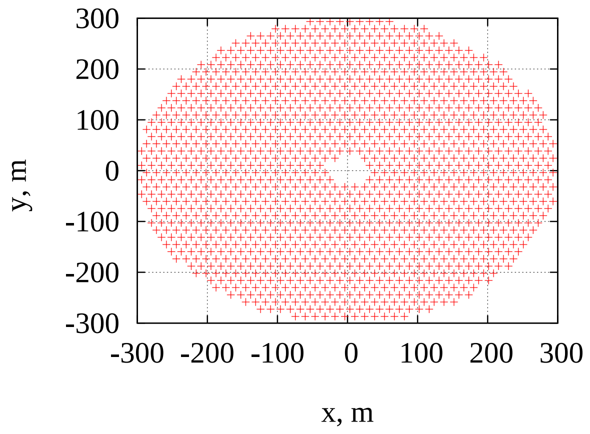

FIG. 5: Solar field

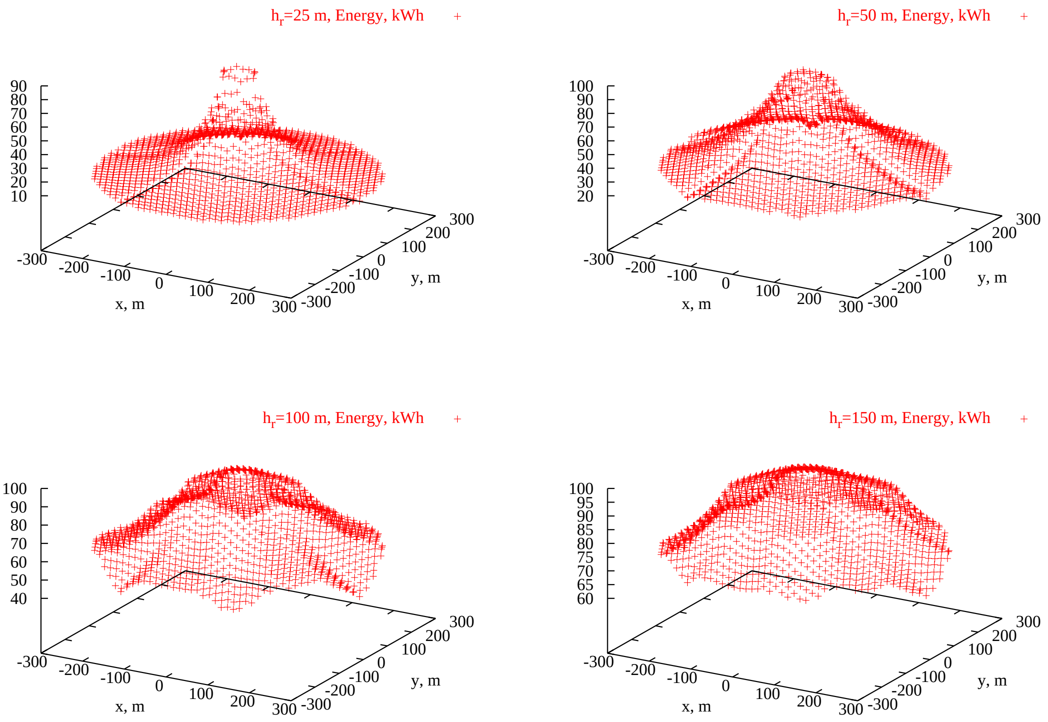

FIG. 6: Energy collected from the solar field at different heights of the receiver . Sun is at $\alpha_z = 0^o$ and $\phi = 0^o$ and DNI=1 kW/m^2.

the direct normal irradiation (DNI) satellite data are given from DRL (the German Aerospace Center) and correspond to solar irradiation at each hour of the year 2005 at 20 m of height from the ground. The mounthly DNI and the mounthly effective DNI$_{eff}$= DNI cosα_z are reported in

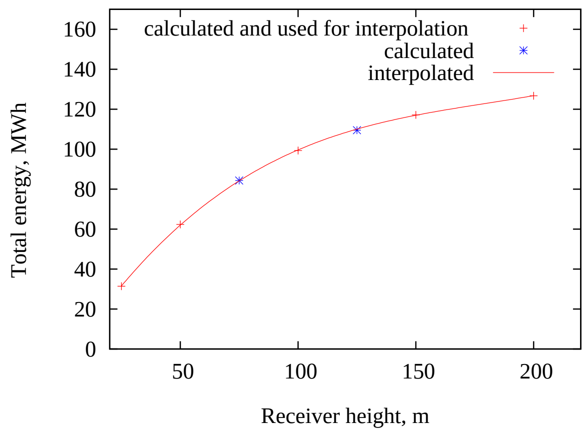

FIG. 7: Total energy collected at the receiver as a function of the receiver height. Calculated points (+) have been interpolated with the polynomial $f = -8.7553 + 1.8319x - 0.0091866x^2 + 1.7081 \times 10^{-5}x^3$. Calculated points (*) lie on the interpolated curve.

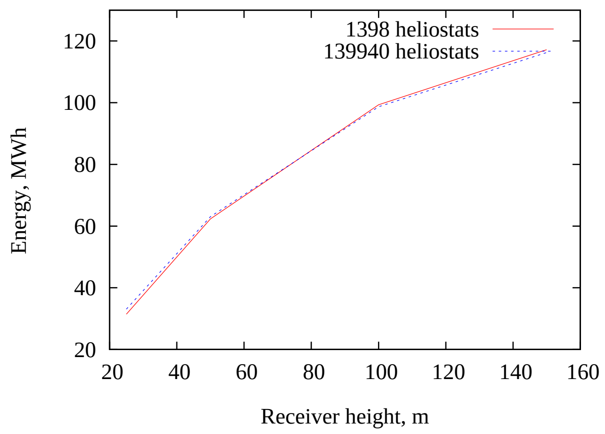

FIG. 8: Comparison between the solar fild with 1398 heliostats of size 10× 10 m^2 with the solar field with 139940 heliostats of size 1× 1 m^2

Fig.9. The annual value of DNI and DNI$_{eff}$ are 1.677 GWh/m^2 and 987 MWh/m^2, respectively.

A series of simulations has been performed both varying the receiver height and the solar field size of the ST plant. In particular, the following receiver heights have been taken into consideration: h_r = 25, 50, 100, and 150 m. For what concern the solar field size, three cases has been considered, all characterized by a circular crown with internal radius of 30 m but external radius, R_{max}, of 150 (case a), 200 (case b), and 300 m (case c), respectively. In each case the solar field has a constant panel density, with a chessboard disposition, which implies a land occupancy of about 50%. Characteristics of the ST solar fields are summarized in Tab. I.

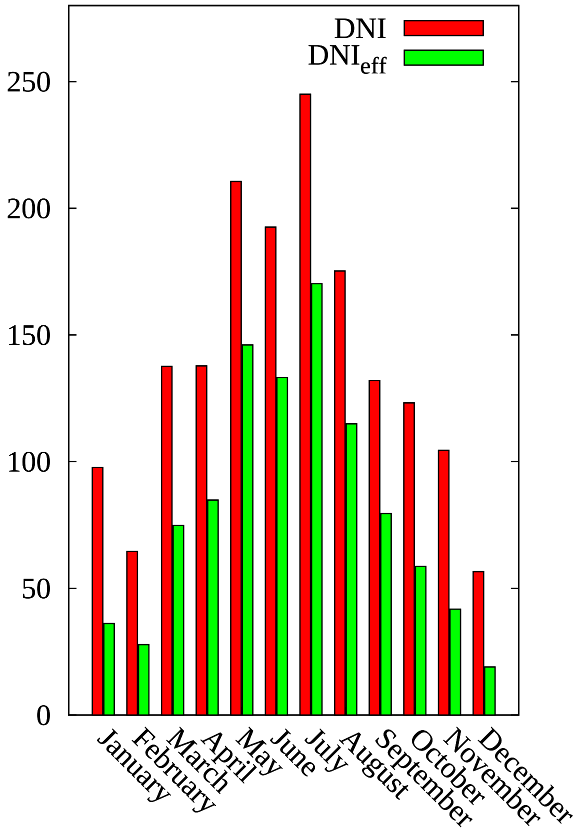

FIG. 9: Mounthly direct normal irradiation, DNI and effective DNI_{eff} =DNIcos(α_z), at Cagliari during 2005

TABLE I: Characteristics of the solar tower field.

	case a	case b	case c
Number of heliostas	337	612	1398
Total heliostats surface, m^2	33700	61200	139800
Total land occupied, m^2	67858	122836	279915

The performance of the considered ST fields have been compared with those of parabolic trough (PT) fields of comparable mirrors sizes. In particular, the calculations of the performace of the PT plants have been done by using the numerical code SUNCOLLECTION.f, written in Fortran77, and developed in our laboratory[1]. Each PT field is analyzed by varying the distance between adjacent rows of mirrors, and compared with the ST system with the same mirrors size. In particular

TABLE II: Characteristics of the parabolic trough solar fields.

To compare with	ST case a	ST case b	ST case c
Number of mirror rows	20	36	37
Mirror opening, m	5.45	5.45	5.45
Mirror lenghts,m	309.175	311.93	693.28
Total mirrors area, m^2	33700	61200	139800

the following distances, d, have been considered: 5.5, 10, 15, 20, and 50 m, respectively. Moreover, the rows of mirrors are always oriented along the North-South directions, which corresponds to optimal disposition. The main characteristic features of the considered PT fields are reported in Tab.II.

The results are reported in the histograms of Fig. 10, where mounthly collected energies are compared between ST plants with different receiver heights and PT plants with increasing distance between adjacent rows.

Finally, comparisons of the annual collected energies between ST and PT systems are shown in Fig. 11, where for the three cases of interest, energies are expressed as function of the receiver heights, h_r, (for ST plants) and distance, d, between adjacent rows of mirrors (for PT plants). The land coverage of the PT plants is also plotted as function of d.

The results of this analysis show that for ST plants of small size, let see, for example, the case with $h_r = 50$ m and $r_{max} = 150$ m, annual collected energy is comparable with that of PT plants of the same mirror size and $d = 15$ m, but the land occupancy of the ST plant is about 50% while that of the corresponding PT plant is about 28%. A comparison between ST and PT plant performance as function of the solar field area is resumed in Fig. 12, where the cases of ST plants with $h_r = 50$ and 100 m are compared with the cases of PT plants with $d = 10$ and 15 m, respectively.

C. Test case 3: Effect of the ground inclination on the solar tower performances.

Finally, it has been studied the effect of the ground inclination on the performances of a solar tower field. Starting from the disposition of heliostats presented in the previous subsection, different inclinations has been taken under consideration for the heliostats located at NORTH

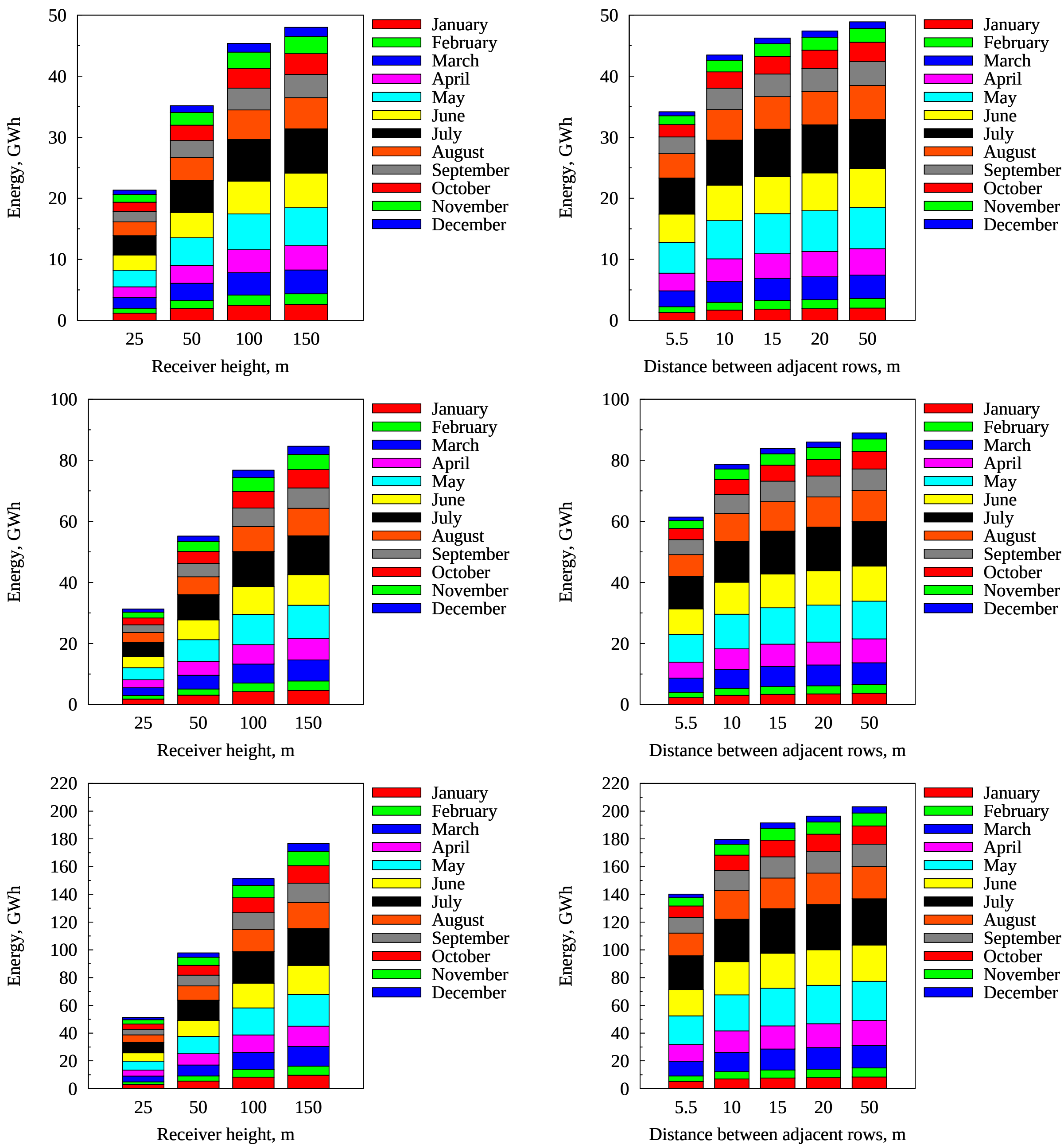

FIG. 10: From the top to the bottom: case a: R_{max}=150 m, case b: R_{max}=200 m and c: R_{max}=300 m. Comparison of mounthly collectd solar energy between ST and PT plants. Different cases are considered varying the receiver height of the ST system (on the left) and the distance between adjacent mirror rows of the PT system (on the right).

with respect to the receiver. In particular, the following ground slopes has been considered: $\alpha_g = 2.5^o, 5^o, 7.5^o$ and 10^o, as shown in Fig. 13, where the dimension of the field of heliostats is that of the case a of the previous subsection, that is, of a circular crown, with maximum distance

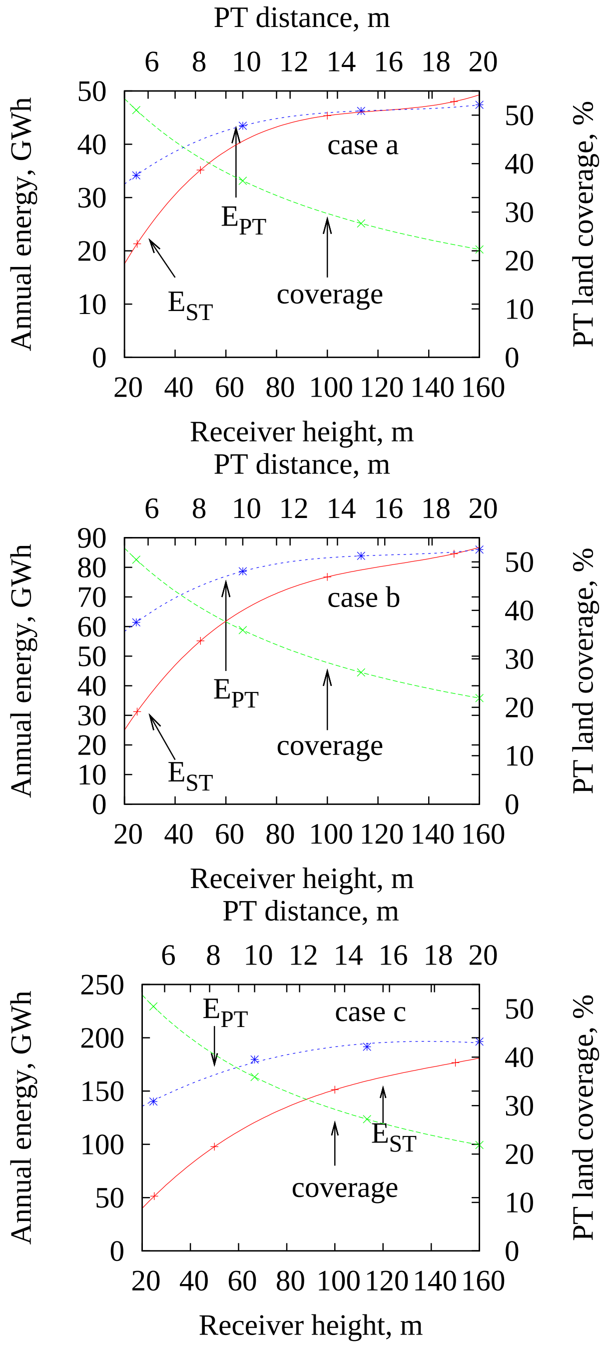

FIG. 11: From the top to the bottom : cases a, b, and c. Comparison of annual collected energy between ST (red line) and PT (blue line) plants, as function of the receiver height and distance between adjacent rows of mirrirs, respectively. Also, land coverage (green line) as function of the distance between adjacent rows of mirrors is shown.

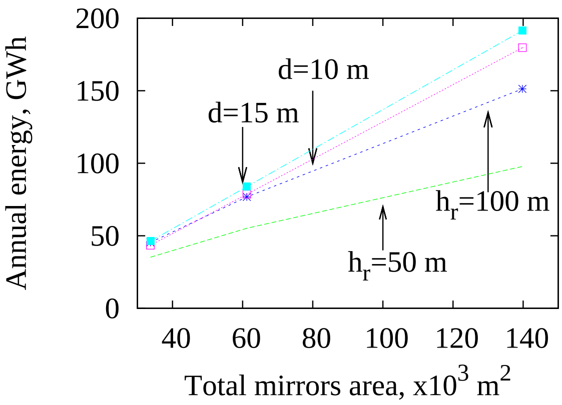

FIG. 12: Comparison between ST and PT field as function of the mirrors area. In paricular, the cases with h_r=50 and 100 m of the ST system have been compared with the cases with d=10 and 15 m of the PT system.

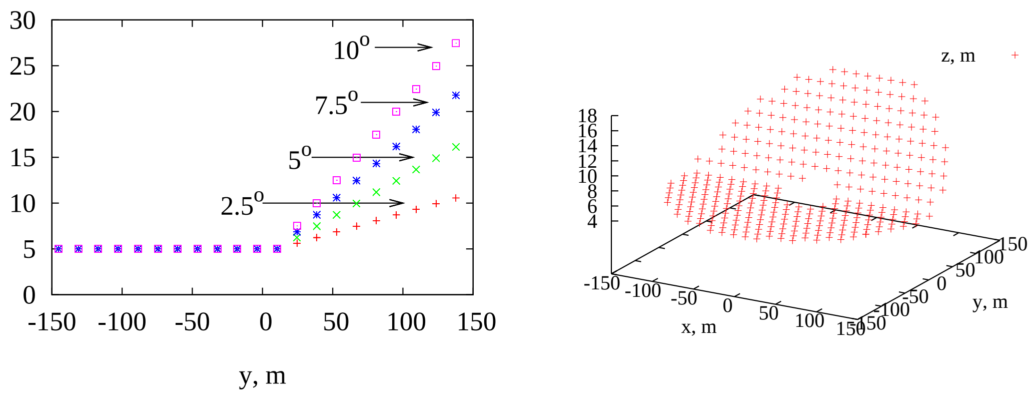

FIG. 13: On the left: heliostats disposition at differents ground slopes. Only the heliostats located at NORTH with respect to the tower are considered on an inclinated ground plane. On the right: the full solar field for the particular case with $\alpha_g = 5^o$.

of 150 m from the receiver.

Of course, the heliostats located at SOUTH with respect to the receiver are, in general, disadvantaged, because they are always between the sun and the receiver, and therefore they are characterized by greater cosine effects with respect to those located at NORTH, and each dispositions of such panels on an inclined ground is less favorable than that on an horizontal plane.

The results of the calculations performed considering a ST system with the receiver height $h_r = 50$ m are shown in Fig. 14, where it appears that the effect of the ground slope is sensible

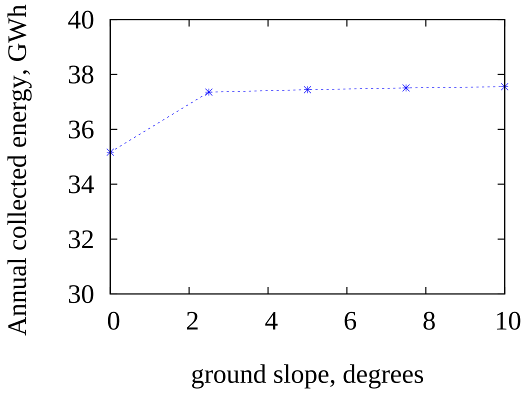

FIG. 14: Dependence of the annual collected energy on the ground slope with a receiver height of 50 m.

passing from $\alpha_g = 0^o$ to 2.5^o and then tends to reduce as the ground slope is further increased.

IX. CONCLUSIONS

In the present report the mathematical model implemented in the numerical code SOLAR-TOWER has been presented. The code is able to compute the energy collected by a field of squared heliostats and reflected to a central receiver system for any arbitrary panels disposition, and geographic site, comprised between the North Pole and the Tropic of Cancer.

Comparisons between Solar Tower systems and parabolic trough systems, obtained running the code SUNCOLLECTION, also developed in our laboratory are shown. The results indicate that ST plants of small size can be favourable with respect to PT plants because of a better exploitation of the land.

The exploitation of ground slopes has also been considered with the final consideration that if the heliostats of the ST systems located at NORTH with respect to the tower are disposed on an inclined plane a further energy recovery is possible.

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[1] E. Leonardi, **Calcolo Dell'effetto Coseno e Delle Ombre su un Campo di Specchi Parabolici Lineari** CRS4 internal report, Number 08/07 (2008).